

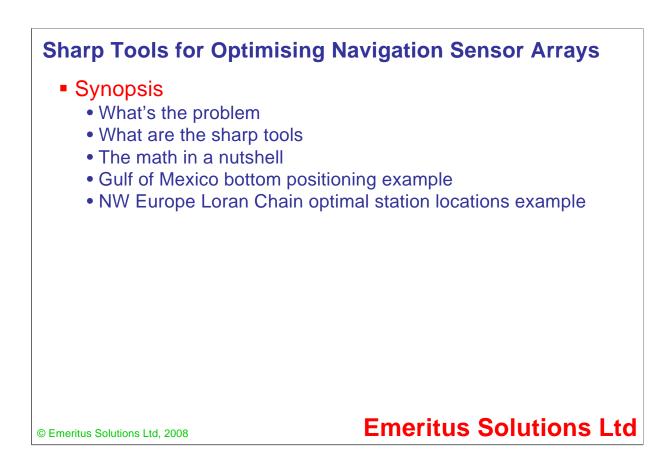
Abstract

For most navigators DOP is a given, and activities are tailored to it. However some navigators, who deploy sensor networks, "make their own DOP". Undersea navigation is a domain in which this paradigm is increasingly important, similarly location services make every radio mast a navigation sensor. This increase in the volume of sensor network design requirements requires sharper tools, which can inform design decisions by less expert users. To do so, they must be better not only graphically but also in the metrics they present to inform the designer's decisions.

Significant factors in sensor network design include: the required areal and vertical coverage, the quality of coverage required, system constraints (e.g. poor clock accuracy), the topography or bathymetry, and certainly at sea the propagation medium's characteristics. Traditionally in designing sensor networks an expert navigator / surveyor has considered these explicit factors together with a substantial body of tacit knowledge, e.g. the often catastrophic reduction of position accuracy outside the sensor array. Beyond managing the data, tomorrow's tools need to offer a sharper critique of performance than a washed out spatial DOP plot.

Plots of DOP are the de facto standard for the "quality" of a position fix. However, DOP values are only as valid as the assumptions which define the problem analysed. Questionable, common assumptions in undersea navigation include: precise knowledge of the propagation velocity and the absence of clock biases. DOP plots generally indicate modest degradation at the edge of sensor arrays. However, the practical navigator knows that without a fair measure of symmetry in the user – sensor geometry a fix is, in the absence of perfectly corrected measurements, very likely to be biased. The metrics used by tomorrow's tools must address this limitation.

This paper will approach the sensor network synthesis problem from analytic and practical perspectives, propose metrics which we consider powerful compliments to DOP, and present comparative examples. The examples presented will range from the tutorial, through the underwater realm and European Loran coverage. The presentation will use graphical tools which permit the designer to work with tens of sensors and the constraints imposed by DTMs (digital terrain maps).



Synopsis

The problem of central interest is the design of positioning sensor layouts for use near the sea bed, typically to support oil and gas development activities. The difficulties which frustrate the trivial solution of an equilateral pattern are the bathymetry and vertical refraction of the lines of sound.

The sharp tools are based on several numeric performance measures. The crucial factors in obtaining a good metric are considered to be: the selection of quantisation thresholds and colour schemes, and the physical insight provided by the metric. The measures presented include DOP, support (how well surrounded a user position is by sensors) and sensitivity (how symmetric the sensor positions are).

Additional, sharp tools are OpenGL, GPUs (Graphics Processing Units), Quaternions and all the other facets of computer graphics. Built on these are the bathymetry model (DEM / DTM), the joint visualisation of bathymetry and performance metric, and the interactive viewing of the data. The sharpest tool of all is of course Mk 0 wetware when presented with all the data in a comprehensible format – the human task being sensor location optimisation.

The necessary mathematics is presented "in a nutshell", to introduce terminology. The paper's appendices contain a fuller version of the problem formulation and analysis.

The principal example used is the design of a bottom positioning sensor layout for the Gulf of Mexico. The bathymetry is from public domain sources and has a coarse resolution of 100 x 100 m horizontally. While this blockiness causes the design to be dominated by quantisation artefacts, the power of the tools in identifying issues and more importantly potential solutions is plainly demonstrated.

The second example presented is an analysis of alternative locations for UK / Irish Loran stations. Here without the requirement for height data, the Z dimension can be used to display metric values.

Sharp Tools for Optimising Navigation Sensor Arrays Synopsis What's the problem What are the sharp tools The math in a nutshell Gulf of Mexico bottom positioning example • NW Europe Loran Chain optimal station locations example Emeritus Solutions' Expertise Technical Consultancy Navigation Solutions : Algorithms and Analysis Underwater Acoustics : Algorithms, Analysis & Materiel Digital Signal Processing : Algorithms and Mechanisation Computer Systems and Software Tools System Development and Verification **Emeritus Solutions Ltd** © Emeritus Solutions Ltd, 2008

Emeritus Solutions' Expertise

Technical Consultancy is frequently the mechanism by which Emeritus Solutions' expertise is initially accessed. Specimen subject domains are: Oil and Gas, Sonar, Navigation, Communication and Computing.

Our navigation expertise is based on a profound understanding of underwater tracking systems. Specifically, Emeritus Solutions has sharp tools for the analysis and implementation of trilateration tracking solutions. Our tools for the analysis and synthesis of tracking systems while tailored to sub-sea requirements are applicable to the analysis of GPS, Loran-C and mobile phone systems. Additionally we also have extensive experience of GPS and highly accurate clocks.

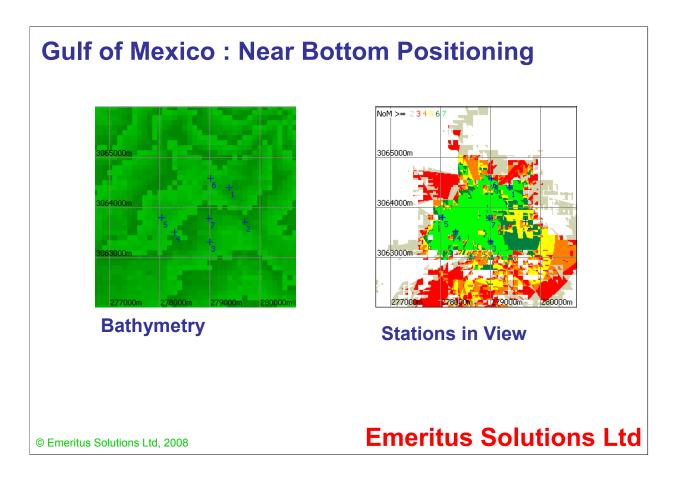
A profound understanding of underwater acoustics, sonar systems and algorithms is available. Our expertise encompasses a broad spectrum from propagation, through sonar equation evaluations, to system implementation and grooming.

In DSP (Digital Signal Processing) Emeritus Solutions has extensive expertise in algorithm design and implementation for time series processing. This expertise encompasses filter analysis, design and mechanisation, complex signal (I+Q) representations and heterodyne design, spectrum (fft / narrowband) and correlation (broadband) processing, non-linear processing, beamforming, signal measurement and of course signal conversion, data transmission and A to D & D to A conversion.

In Computer Systems our expertise extends from machine code to productivity tools.

Equally, system development and verification whether from a clean or well smudged sheet of paper are core competences with expertise available to identify current and necessary conformance to requirements. Typical systems will commonly include a mixture of PCs, embedded processors, microcontrollers, FPGAs and Digital Signal Processors (DSPs).

Finally, in Electronics and everyday engineering our expertise is of course extensive.



Bathymetry

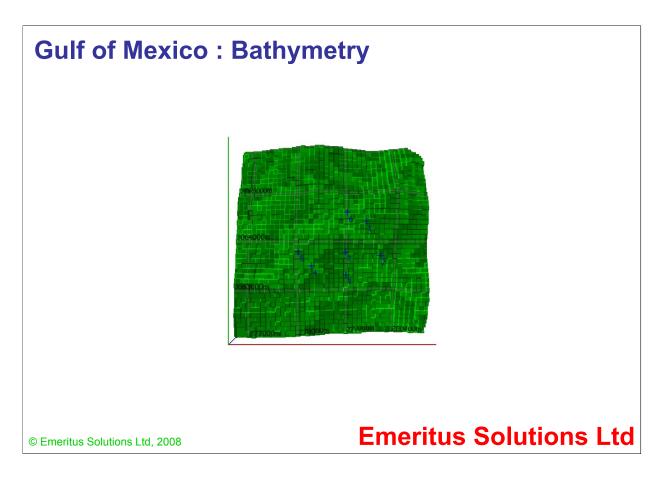
The specimen data plotted above is a small extract from the US national Geophysical data Centre (NGDC) 3 Arc-Second Coastal relief Model dataset. The resolution used on the computer is 100 x 100 meter cells, on a UTM grid (UTM zone 16). The XY resolution is of course much coarser than the 2 to 5 m X & Y resolution of a commercial survey. Similarly, the depth resolution and accuracy are probably no worse than a commercial survey, but the large blocks result in slopes becoming steps. In any case, as we shall see, it is good enough for a tutorial example.

The difficulty of visualising depth from a two dimensional representation, even with greenscale contouring. The use of interactive 3-D graphics, as we shall see, effectively resolves this issue.

Stations in View

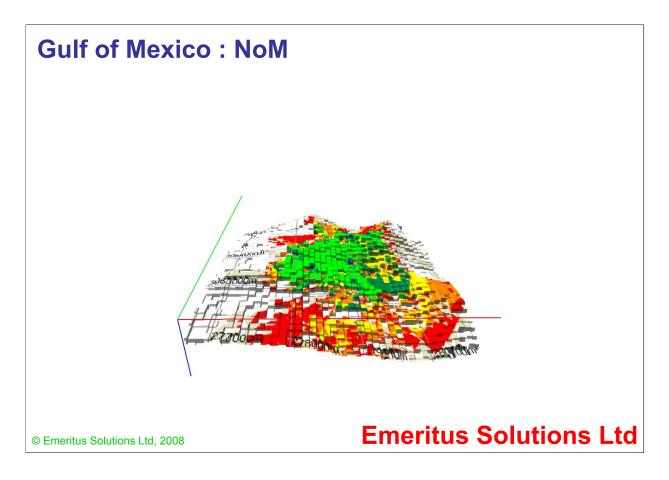
The simplest metric used for positioning coverage / quality is the number of stations in view at each user position. The plot depicted above is based not simply on cookie cutter ranges, which can be very misleading in a near bottom environment, but on modelled visibility for 10 m station and 5 m user height accounting for bathymetry and refraction.

While it is easy to see where coverage is good, poor and absent. It is very difficult to infer why there are problems and more significantly, what the potential solutions may be. Again, as we shall see, the use of interactive 3-D graphics, effectively resolves this issue.



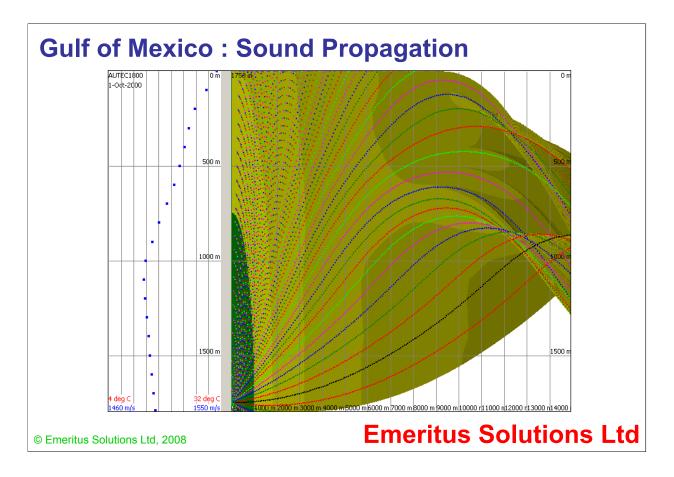
Bathymetry - A 3-D Video Clip

Playing the .avi file embedded in the PowerPoint presentation provides a definite insight into the terrain. Interactive interaction on a laptop computer provides complete insight into the lie of the land.



Number of Measurements - A 3-D Video Clip

Playing the .avi file embedded in the PowerPoint presentation provides a definite insight into the relationship between visible stations terrain. Interactive interaction on a laptop computer provides complete insight into which terrain features cause difficulties and what station relocations could be beneficial.



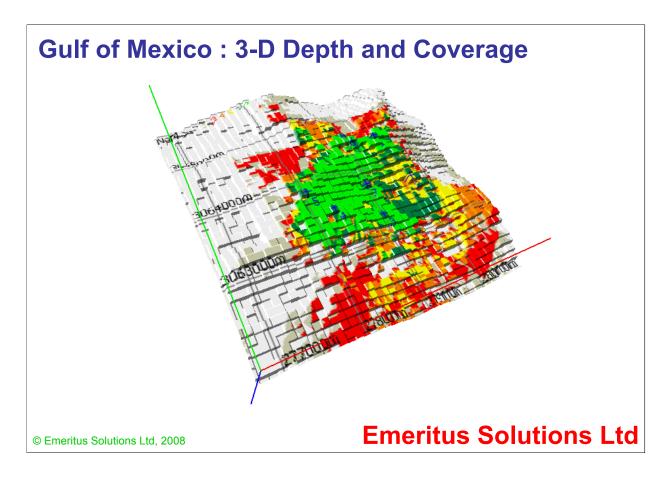
Sound Propagation

Sound propagation and acoustics are always central to underwater acoustic positioning. The list of potential issues is extensive, encompassing attenuation, reverberation, noise levels, multipath and most importantly refraction. The speed of sound of seawater varies vertically and to a lesser degree horizontally. The effect of the vertical (speed of sound) gradients is to refract the line of sound.

Generally the presence of a line of sound between any two points in the water column is determined by path occlusion due to refraction. The specimen Sound Velocity Profile (SVP) for the AUTEC range off Andros Island in the Bahamas, taken from the internet and depicted above, is typical of a warm, deep ocean; e.g. Gulf of Mexico, offshore Nigeria, and offshore Brazil.

For transducers located near the sea bed and the associated well heads and production infrastructure, the dominant feature is the deep isothermal layer, located in this example below ~1100 m. The essential feature of the deep isothermal layer is the constant positive gradient of ~0.016 m/s / m. The variation with location, if the deep isothermal water assumption holds, is negligible; generally less than ~0.001 m/s / m.

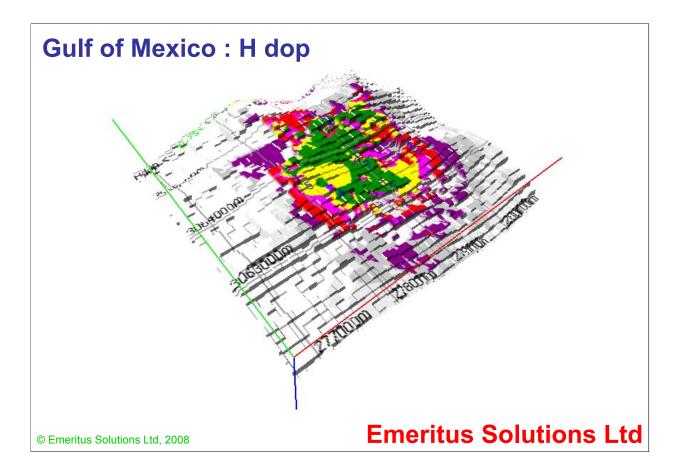
The essential effect of this characteristic is upward refraction of sound. Consequently, there is a bottom shadow zone, the maximum range being proportional to the square root of station height above the topology. Which means that ranges are generally refraction not noise limited. Additionally, the sagging line of sound can be obstructed by intermediate terrain.



Candidate Metrics : Number of Measurements (NoM)

We shall now compare the candidate performance metrics which can be used when preparing station layouts. To make a comparison we present, again, the NoM data for our candidate layout.

Perhaps the most important point to appreciate concerning NoM is that it is a necessary, but not a sufficient condition for satisfactory system performance. Having an adequate number of stations in view does not ensure that they have an adequate geometry. Obviously, in the hands of an expert, NoM taken with rules of thumb – such as only work inside the array – provides a good 1st order appreciation of system performance.



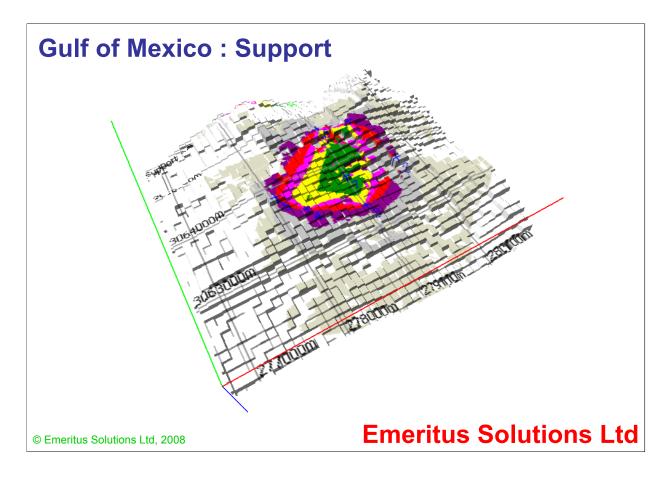
Candidate Metrics – Hdop (Horizontal Dilution of Precision)

Hdop is the classical metric for tracking system performance, the ratio between position noise and range measurement noise. It is broadly an honest metric, except that the analyst's assumption of whether the system knows or estimates the time of emission produces radically different results. Also, in common with all DOP values, it's physical correlates are well hidden in the matrix formulation from which it is derived.

A brief exposition, together with references to the literature, of the geometric interpretation of DOP as the inverse of the projected length / area (onto the appropriate line or plane) of the n-gon formed by the direction cosines from the user to the sensors is provided in an appendix to the paper.

The colour coding used is based on the formula $2 / \sqrt{N}$, where N ranges from 7 (low dop of 0.756) to 1 (very poor DOP of 2.0). That formula, derived by Levanon, is for the Gdop at the centre of an N-gon. Consequently, the DOP thresholds have a well motivated physical interpretation as an equivalent number of optimally distributed sensors.

That coverage outside the array is available is unsurprising, as this analysis assumes that time is known throughout the system. This could reflect the use of synchronised (Rubidium) clocks or of two way range measurements.

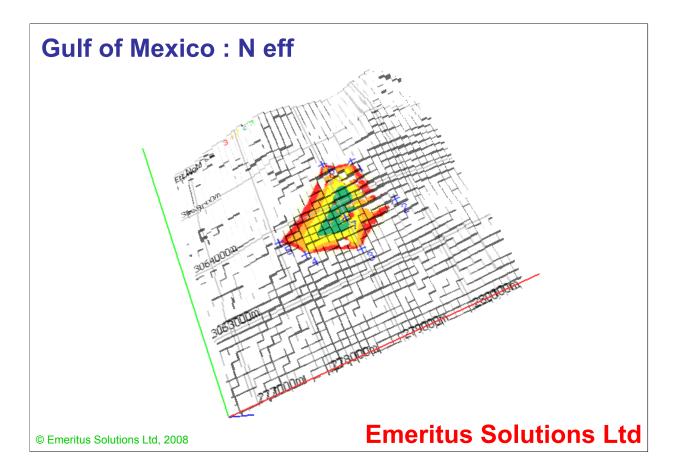


Candidate Metrics – Support

Support, or more precisely horizontal support, is a metric based on the projected area of the direction cosines of the measurements normalised by the area of a unit circle. Self evidently, this metric reflects how well a user position is surrounded by stations. Consequently, the metric is always low outside an array of stations.

The metric is of course analogous to the projected area of the direction cosines mentioned in the previous section. Normalisation of the metric is straightforward with 10% slices being taken from 20 to 80 %. Pedagogic examples of the support achieved by standard array geometries are included in the paper's appendices.

The restriction of coverage to inside the array is unsurprising. The bald patch to the East of the area corresponds to the Hdop analysis.

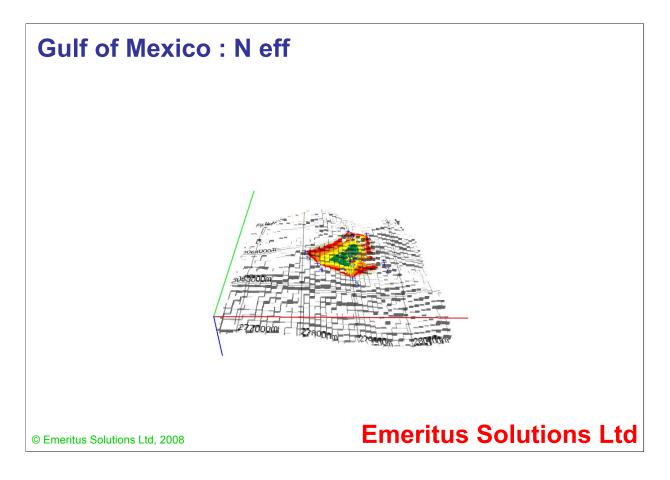


Candidate Metrics – Number of Effective Measurements (N eff)

N eff, the number of effective measurements, is a pessimistic metric based on the assumption that time is not well known. N eff is in fact equivalent to Tdop, is related by the 2 / \sqrt{N} formula and when derived by using a partitioned matrix approach to the time terms yields considerable insight into the geometric origins of Tdop / N eff. The paper presents a derivation of N eff.

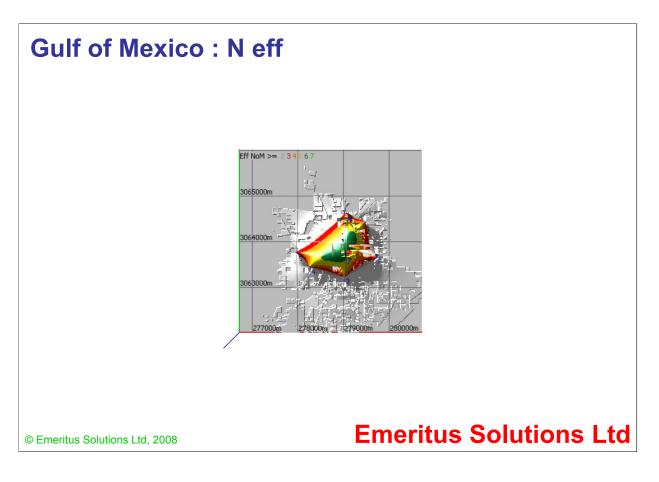
Obviously, the assumptions used in deriving N eff correspond to the rule of thumb to "work inside the array". This approach does of course provide robustness, not only to the need to estimate time, but also to biases due to an imperfect knowledge of the propagation velocity. In essence, particularly in the centre of the array, the errors cancel.

Again the poor coverage to the East is indicated by the metric. The colour coding / thresholds used are the self evident DOP / NoM thresholds.



N eff : .avi clip

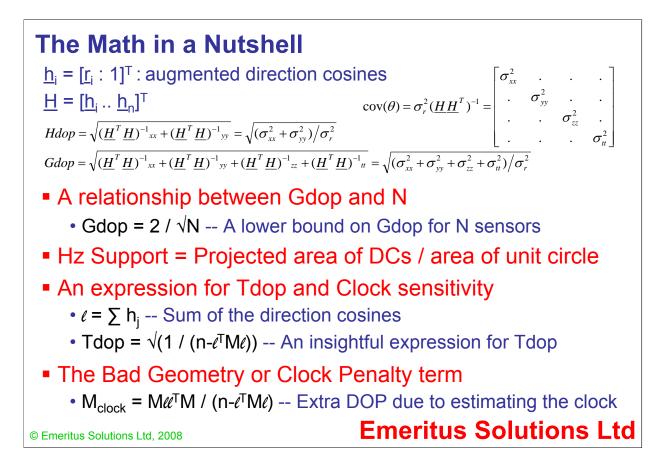
This .avi clip displays the effectiveness of 3-D interactive graphics in displaying 4-D (X, Y, Z & metric) datasets. The lie of the land can be readily appreciated, as can the problem areas and the scope for mitigation.



N eff – X Y Metric plot : .avi clip

The height dimension can of course by used to depict the metrics value, rather than bathymetry. This presentation provides additional insight into the scale and nature of changes to the metric's value. The height is plotted in units of 1 / dop using the usual conversion from N eff. The blue reference line is of unit length.

In this example, it is evident that the area to the east is characterised by some catastrophic failures in coverage, almost certainly due to the simultaneous masking of several stations.



The Mathematical Model

The standard linearised least squares set up is employed with the, user centred, direction cosines describing the measurement set up. Also, the usual simplifying assumptions concerning measurement noise and weighting matrices to eliminate all factors but the H matrix (of the direction cosines).

The well known solution to this model, which is described at greater length in the paper and it's references, yields a covariance matrix (of estimated position errors) which when normalised by measurement error is $(HH^{T})^{-1}$. Obviously, the diagonal terms provide the covariance terms for estimates of the X, Y, Z and (if required) T elements of the position vector. Equally, the expressions for Hdop, Gdop, etc follow naturally.

DOP and N

The relationship between Gdop and N, Gdop = $2 / \sqrt{N}$ at the centre of an N-gon due to Levanon provides a very useful conversion formula which we use to equate the graphical representations (size and colour) of dop and number of measurements.

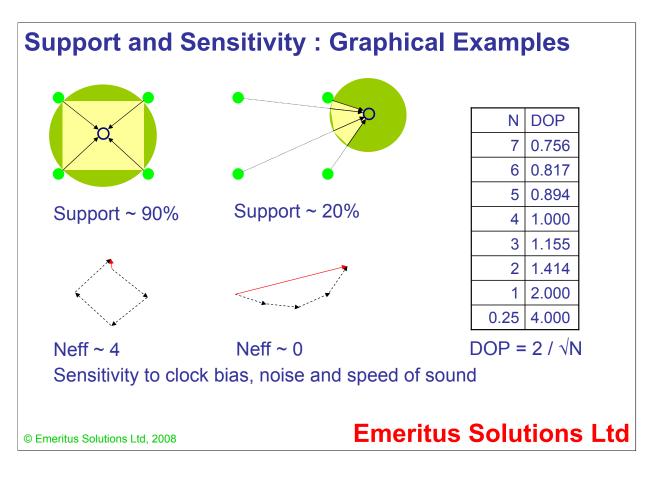
Hz Support

Is a useful metric, with a firm geometric basis, which relates sensor geometry and performance. Simple examples are given on the next slide.

Clock Sensitivity

Sensitivity to clock effects are very important in real-world problems. This may reflect the requirement to estimate the clock epoch – the pure Tdop problem. Or, it may reflect the need to be robust to a poor knowledge of time at the user or sensors, or a poor knowledge of propagation velocity.

The results summarised here show the mathematical underpinning of the rule of thumb that one should "work inside the array". At its simplest, if the observation geometry (in terms of the direction cosines) is (nearly) symmetric the sum of the DCs is minimal and the clock epoch and XYZ position can be estimated with greatest accuracy.



Support

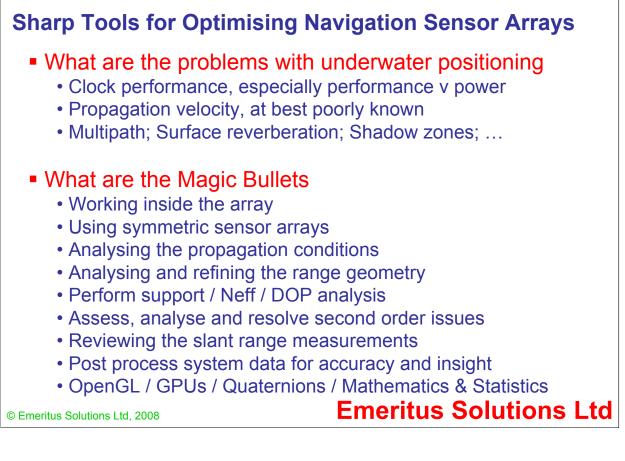
The two examples of support show how it relates to the geometry of the direction cosines used to compute a position fix. The advantage of employing multiple, enclosing sensors being self evident.

Sensitivity

The two examples of sensitivity show very clearly the advantage of a balanced sensor geometry. In simple terms beyond providing accurate estimates of emission times, any systematic errors in, for example, propagation velocity will cancel.

DOP = 2 / \sqrt{N}

A tabulation of the standard values used as thresholds in the examples presented is, as the reproduction on the plots can be difficult to discern, provided for reference. It is interesting to note that dop values above ~1.2 are actually quite poor. Perhaps we should be cautious rather than accepting of large DOP values.



Problems with Underwater Positioning

The primary (generic) problems with underwater positioning are knowledge of time, which generally has an inverse relationship to power consumption and cost. Knowledge of propagation velocity, which is generally poor, accuracies below 1 m/s not in instrumentation but in measurement practice seem commonly unattainable. Then there are all the acoustics issues, both obvious and second order. Finally, the bathymetry is of course central to the design of near bottom positioning systems.

Magic Bullets

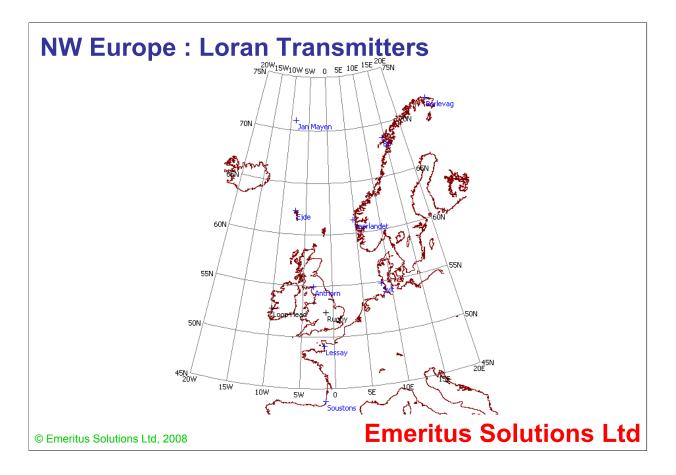
In the context of the sharp tools presented in this paper the origins of their requirements can clearly be seen in the list of magic bullets. The last entry in the list is of course the list of magic bullets required to implement those requirements.

The pervasive availability of inexpensive graphics engines (GPUs) with good APIs (OpenGL) is fundamental to the implementation of the interactive graphics required to provide the user with sharp graphical tools for analysing candidate designs.

The availability of a range of metrics which provide different, complimentary insights into the "optimality" of candidate designs is of course essential. With Nom, DOP, Support and Neff we have the desired repertoire of metrics each associated with differing geometric insights or rules of thumb.

The underpinning magic bullet is of course the Queen of Sciences, Mathematics. Statistical and Mathematical tools being absolutely fundamental to the modelling, solution and analysis of this problem.

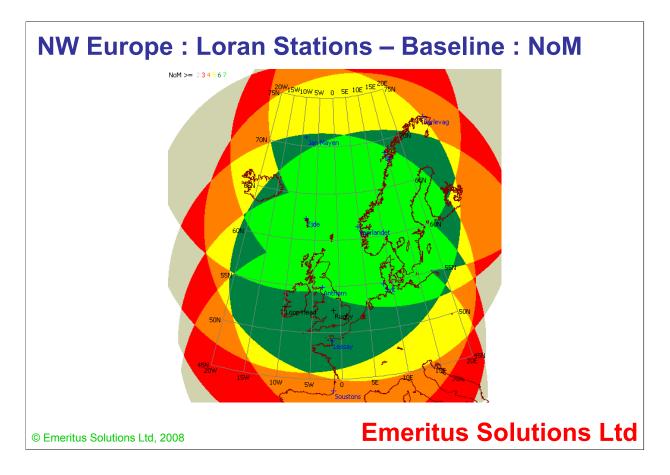
Even the graphics has significant mathematical underpinnings, for example: The need to use quaternions to represent orientations in the 3-D interactive graphics. The use of homogeneous coordinates (from projective geometry) in OpenGL.



NW European Loran Transmitters

Active stations are indicated in blue, with Rugby which relocated to Anthorn and Loop Head which was not to be in black.

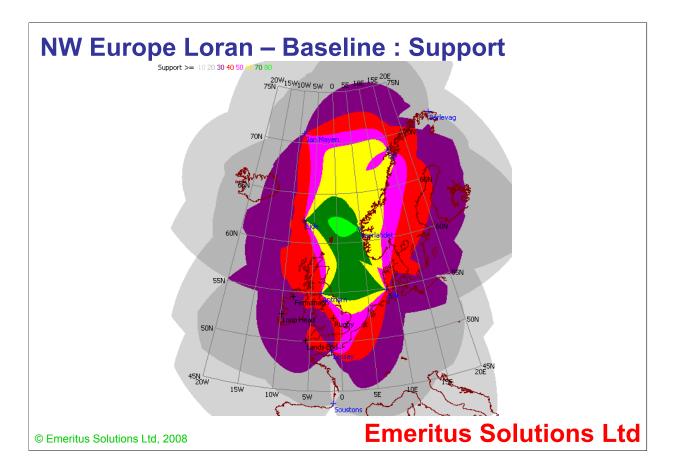
The projection is a Lambert conformal conic with two standard parallels.



NW European Loran : Number of Measurements

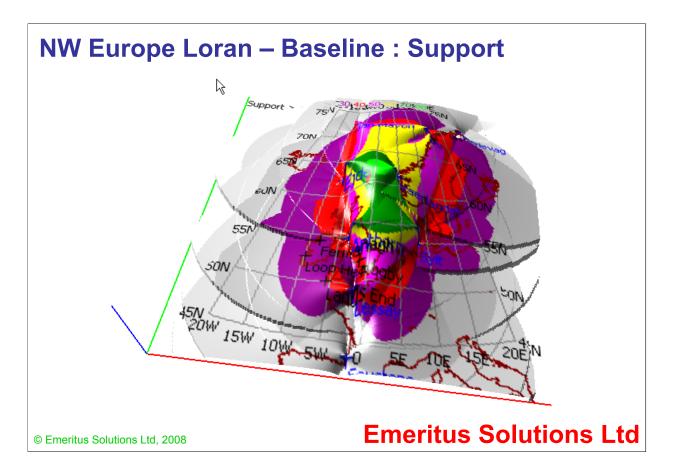
A simple "cookie cutter" analysis of the European Loran stations suggests good coverage.

The "cookie cutter's" radius is 1000 Nmi. Chosen as it represents the limit of reliable ground path coverage, or more specifically the start of unreliable sky wave coverage.



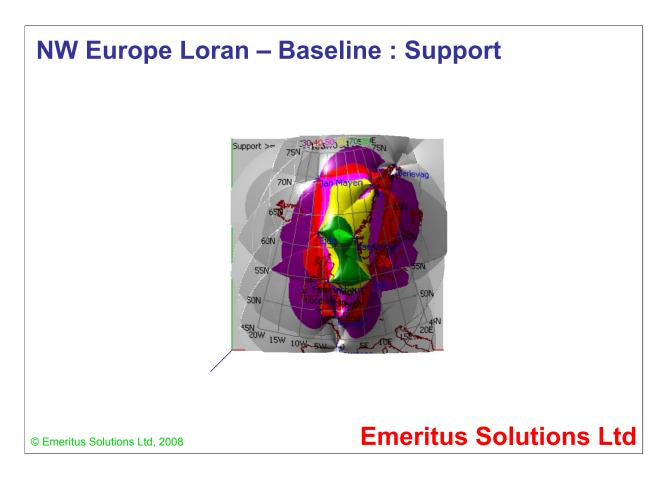
NW European Loran : Support

The support plot for the baseline stations tells a rather different story. UK coverage, especially in the West is not particularly good.



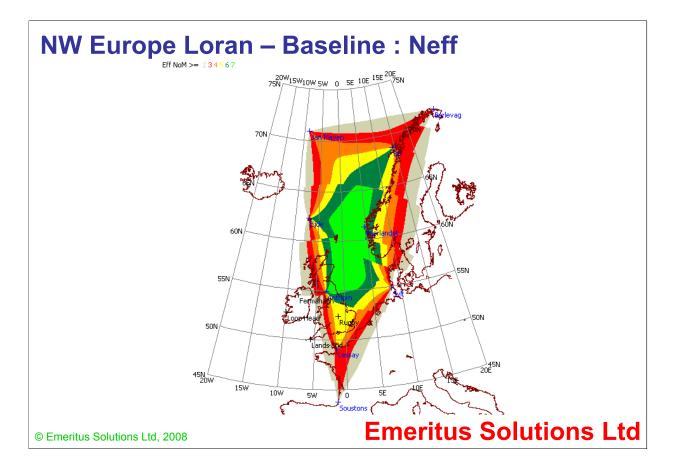
NW European Loran : Support – 3-D

This 3-D view shows very clearly the additional insight afforded by presentation of a metric as a vertical extent. Also, it clearly identifies the negligible performance cost of excluding distant stations from the solution at 1000 NMi – note the slight steps in the surface.



NW European Loran : Support - .avi

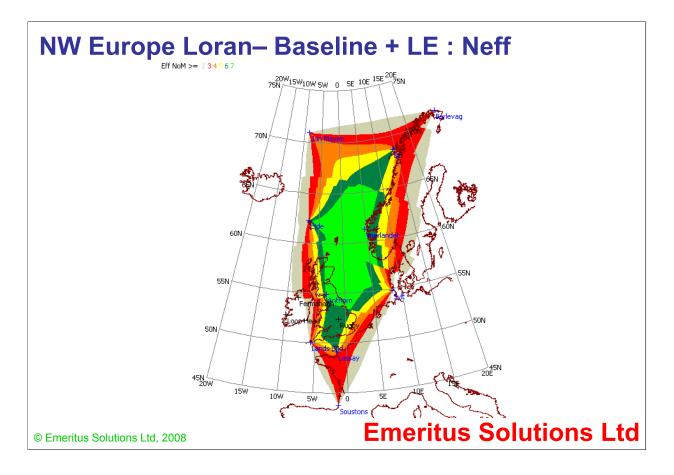
This .avi clip again indicates the insight available from interactive 3-D graphics representations of data.



NW European Loran : Neff

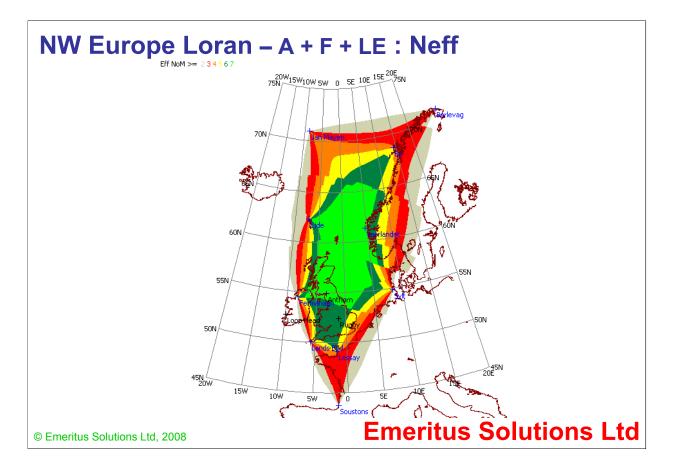
N eff offers a rather more pessimistic view of the coverage. The question Neff is answering is where can one derive time from the system. To obtain an accuracy equal to or better than the range error Neff must be 4 or greater (orange, yellow or green). Obviously, measurements of at least four transmitters (pseudo) time of arrival (to provide 3 time differences) is required. Note that Neff is distinct from the number of observable measurements.

The key insight of this analysis is that good time transfer is only readily performed inside the array. The pre-survey of receiver locations would of course extend the service area slightly to the Neff = 1 contour. Consequently, a station in the West appears highly desirable or time transfer and as time transfer is fundamental to positioning in Loran for positioning also.



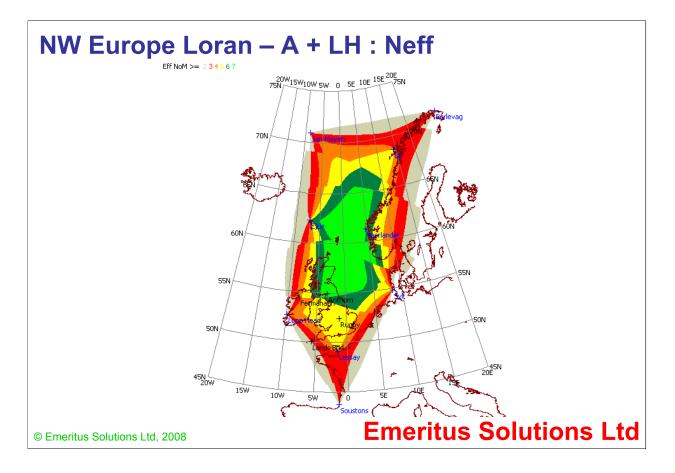
NW European Loran : Baseline + Lands End : N eff

The addition of a station at Lands End, or on the Scilly Isles would self evidently be beneficial.



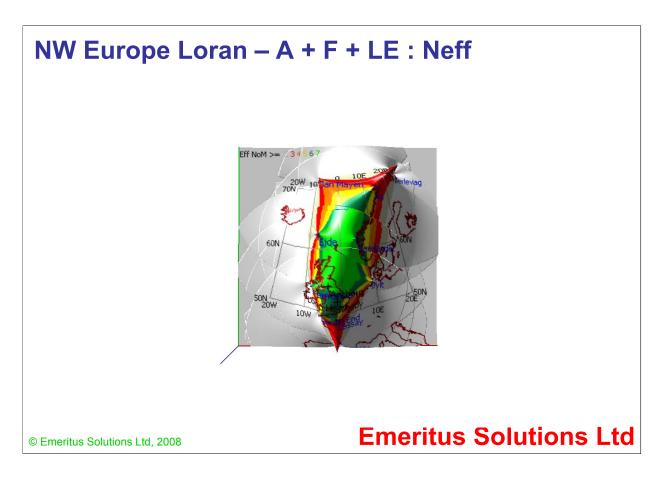
NW European Loran : Baseline – Anthorn +Fermanagh + Lands End

If in addition to a (new) station at Lands End the Anthorn station were to more to Fermanagh the UK coverage would be improved further.



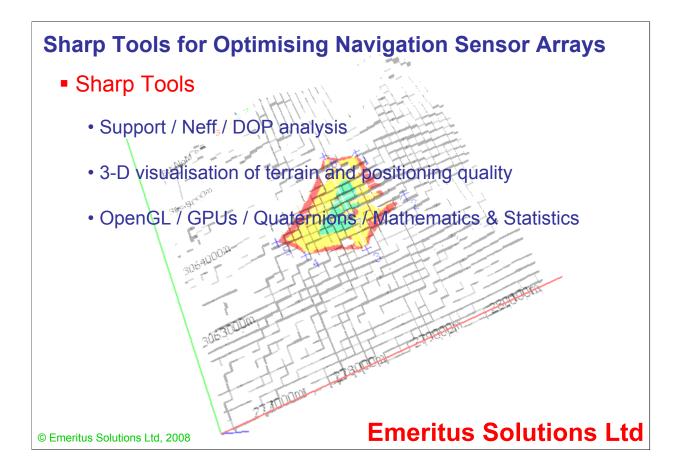
NW European Loran : Baseline – Anthorn + Loop Head

Contrariwise were history to be revisited and Anthorn to be found at Loop Head that one station would afford good coverage of the UK and Eire.



NW European Loran : Baseline – Anthorn + Fermanagh + Lands End : .avi clip

A final demonstration of the sharp tools and an indication of the coverage which could have been provided by Fermanagh and Lands End is provided by this .avi clip.



Support / Neff / DOP / NoM Metrics

With the Support, N eff, DOP and NoM metrics we have a repertoire of metrics each associated with differing geometric insights, rules of thumb or system requirements which offer different, complimentary insights into the "optimality" of candidate designs.

A comprehensive understanding of these metrics and the associated rules of thumb is of course the sharpest of our Sharp Tools.

3-D Visualisation

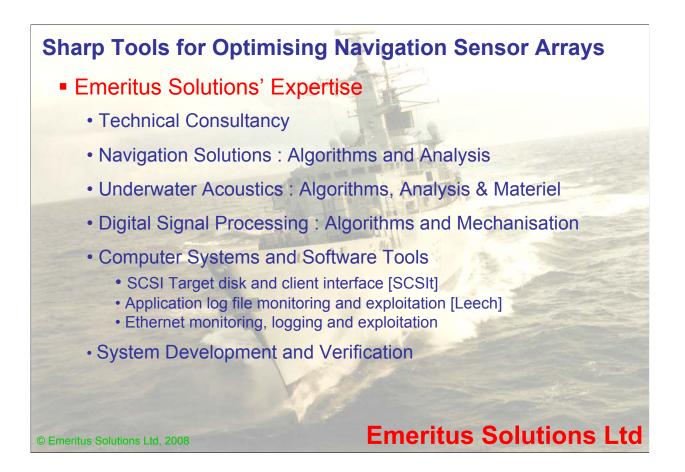
The greatest problem when designing 3-D layouts is that one cannot visualise the terrain. In particular, it is very difficult to appreciate the lie of the land to identify the sources of masking and to evolve robust layouts. The use of 3-D graphics instantly resolves this issue and by colour coding permits some movement into a 4th (value) dimension.

OpenGL, GPUs

The pervasive availability of inexpensive graphics engines (GPUs) with good APIs (OpenGL) is fundamental to the implementation of the interactive graphics required to provide the user with sharp graphical tools for analysing candidate designs.

Mathematics and Statistics

The underpinning sharp tool is of course the Queen of Sciences, Mathematics. Statistical and Mathematical tools being absolutely fundamental to the modelling, solution and analysis of this problem.



Emeritus Solutions' Expertise

Technical Consultancy is frequently the mechanism by which Emeritus Solutions' expertise is initially accessed. Specimen subject domains are: Oil and Gas, Sonar, Navigation, Communication and Computing.

Our navigation expertise is based on a profound understanding of underwater tracking systems. Specifically, Emeritus Solutions has sharp tools for the analysis and implementation of trilateration tracking solutions. Our tools for the analysis and synthesis of tracking systems while tailored to sub-sea requirements are applicable to the analysis of GPS, Loran-C and mobile phone systems. Additionally we also have extensive experience of GPS and highly accurate clocks.

A profound understanding of underwater acoustics, sonar systems and algorithms is available. Our expertise encompasses a broad spectrum from propagation, through sonar equation evaluations, to system implementation and grooming.

In DSP (Digital Signal Processing) Emeritus Solutions has extensive expertise in algorithm design and implementation for time series processing. This expertise encompasses filter analysis, design and mechanisation, complex signal (I+Q) representations and heterodyne design, spectrum (fft / narrowband) and correlation (broadband) processing, non-linear processing, beamforming, signal measurement and of course signal conversion, data transmission and A to D & D to A conversion.

In Computer Systems our expertise extends from machine code to productivity tools.

Equally, system development and verification whether from a clean or well smudged sheet of paper are core competences with expertise available to identify current and necessary conformance to requirements. Typical systems will commonly include a mixture of PCs, embedded processors, microcontrollers, FPGAs and Digital Signal Processors (DSPs).

Finally, in Electronics and everyday engineering our expertise is of course extensive.



Biography

Eur Ing Dr Martin Bishop is a Technical Consultant and the Director of Emeritus Solutions Ltd.

Martin is based in Dorchester, Dorset, UK. His current interests include positioning systems and algorithms, Digital Signal Processing, system implementation and remediation; typically for the Defence and Oil and Gas sectors.

Martin's formative years were spent with the UK Ministry of Defence and latterly QinetiQ in the Portland area. While with UK MoD he was awarded a Personal Promotion to UG6 and while with QinetiQ was a QinetiQ Fellow.

Dr Bishop has wide experience, gained over 30+ years, of:

- underwater tracking range analysis, development and grooming
- torpedo and sonar target development, calibration and analysis
- · torpedo and allied systems analysis and development
- active and passive sonar design and development
- · submarine and surface ship systems and constraints
- digital signal processing, statistics, computers, electronics, ...
- the technical architecture of navigation, torpedo, submarine and ASW systems

Dr Bishop is a Fellow of the Institution of Electrical Engineers

Contact Details

Eur Ing Dr Martin Bishop CEng FIEE Technical Consultant Emeritus Solutions Ltd 22 Herringston Road Dorchester Dorset DT1 2BS +44 (1305) 262806 Tel / Fax Mjd.Bishop@Emeritus-Solutions.com www.Emeritus-Solutions.com