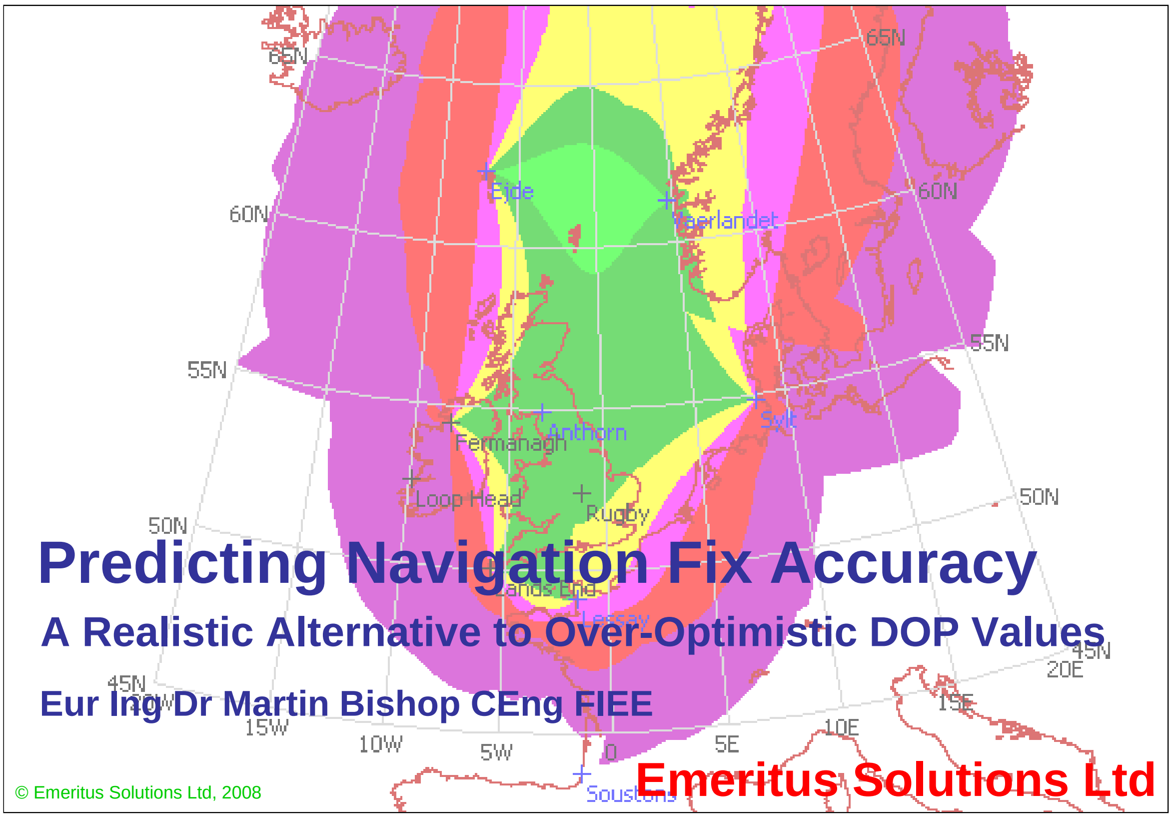

Predicting Navigation Fix Accuracy

A Realistic Alternative to Over-Optimistic DOP Values

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Abstract

DOP (Dilution of Precision) “defines” how good a position fix is. If only it was so simple. Like too many statistical metrics, DOP depends more on the assumptions underpinning the analysis than on the data from which the fix is computed.

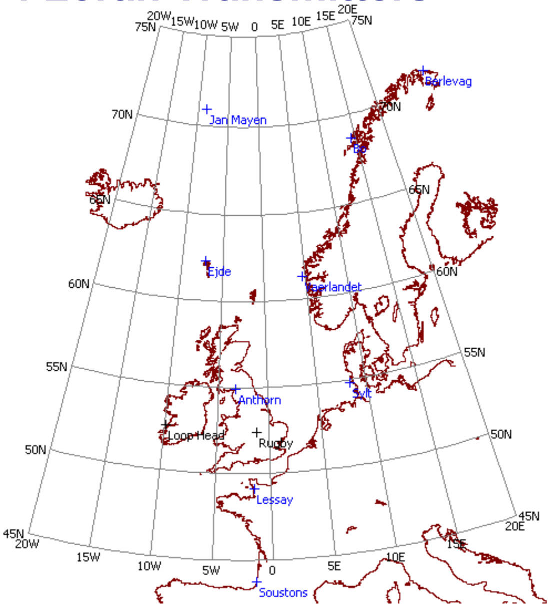
It is well known in the surveying community that symmetric geometries are highly beneficial where accurate, unbiased position solutions are desired. However, where symmetry is lacking, such as at the edge of a tracking range, biased fixes are likely to result. Significantly, the usual DOP analysis and metrics do not identify this performance shortfall and are seriously flawed.

As tracking range deployments become more frequent, performance analysis techniques and tools are increasingly important to mission success. The limitations of the DOP paradigm and an alternative metric which encompasses intuition and is in operational use will be described with comparative examples presented.

Predicting Navigation Fix Accuracy

- Synopsis
  - What's trilateration
  - What's Dilution of Precision (DOP)
  - What's the problem and what's the cure
- Emeritus Solutions Ltd
  - Engineering consultancy
  - Underwater tracking range processing and analysis
  - Underwater acoustics and signal processing expertise
  - Instrumentation of signals, systems and computers
  - Software, hardware and system development

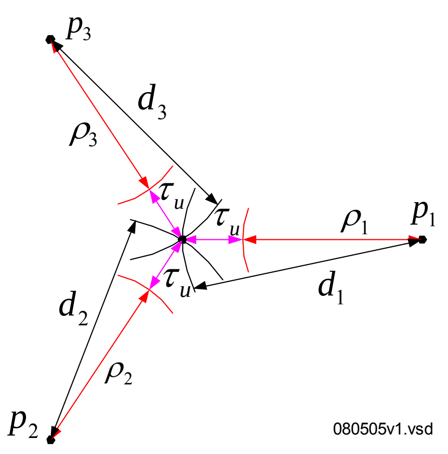
NW Europe : Loran Transmitters



Predicting Navigation Fix Accuracy

- What's the problem with underwater positioning
  - Clock performance, especially performance v power
  - Propagation velocity, at best poorly known
  - Multipath; Surface reverberation; Shadow zones; …
- What are the Magic Bullets
  - Working inside the array
  - Using symmetric sensor arrays
  - Analysing the propagation conditions
  - Reviewing the slant range measurements
  - Analysing and refining the range geometry

True Ranges, Pseudo Ranges and Offsets



$p_i = (x_i, y_i, z_i)$: i_{th} sensor

$u = (x_i, y_i, z_i)$: tracked object

d_i = true range to i_{th} hydrophone

ρ_i = pseudo range to i_{th} hydrophone

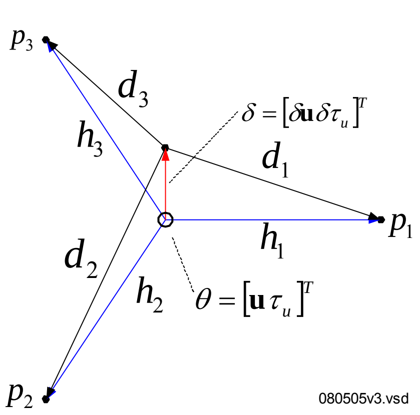
$= d_i - \tau_u$

τ_u = tracked object's clock offset

Δ_{ij} = range differences (aka TDOA)

$= \rho_i - \rho_j$

Position Estimation Framework



$h_i = [r_i : \tau_i]^T \quad H = [h_i \,..\, h_n]^T$

$\underline{h}_i = [r^i : 1]^T \quad \underline{H} = [\underline{h}_i \,..\, \underline{h}_n]^T$

$\underline{h}_i$ = augmented direction cosines

The scalar range equations are

$d_i = \underline{h}_i.h_i - h_i \,.\, \theta$

The measurement residuals are

$\varepsilon_i = \rho_i - d_i = \rho_i - \underline{h}_i.(h_i - \theta)$

In matrix notation

$\varepsilon = \boldsymbol{\rho} - \underline{H}.(H - \theta)$

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Iterative Estimation of Position

For $\delta\theta = [\delta x, \delta y, \delta z, \delta\tau_u]^T$ -- delta position and emission time

with $P = \sigma_r^2.I$ -- i.e. measurement errors are equal and independent

and $W = P^{-1}$ -- i.e. LS weights α 1 / measurement errors

$\varepsilon = \boldsymbol{\rho} - \underline{H}.(H - \theta)$ -- the residuals wrt the estimated position

yeilds $\delta\theta = (\underline{H}^T\,\underline{H}^T)^{-1}\,\underline{H}^T\,\varepsilon$ -- the estimated position update

and cov(θ) -- the estimated positions covariance

$$\mathrm{cov}(\theta) = \sigma_r^2(\underline{H}\,\underline{H}^T)^{-1} = \begin{bmatrix} \sigma_{xx}^2 & . & . & . \\ . & \sigma_{yy}^2 & . & . \\ . & . & \sigma_{zz}^2 & . \\ . & . & . & \sigma_{tt}^2 \end{bmatrix}$$

DOP may now be defined as

$$Xdop = \sqrt{(\underline{H}^T\underline{H})^{-1}{}_{xx}} = \sqrt{\sigma_{xx}^2/\sigma_r^2}\,;\, etc\ for\ Ydop, Vdop\ and\ Tdop$$

$$Hdop = \sqrt{(\underline{H}^T\underline{H})^{-1}{}_{xx} + (\underline{H}^T\underline{H})^{-1}{}_{yy}} = \sqrt{(\sigma_{xx}^2 + \sigma_{yy}^2)/\sigma_r^2}$$

$$Pdop = \sqrt{(\underline{H}^T\underline{H})^{-1}{}_{xx} + (\underline{H}^T\underline{H})^{-1}{}_{yy} + (\underline{H}^T\underline{H})^{-1}{}_{zz}} = \sqrt{(\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2)/\sigma_r^2}$$

$$Gdop = \sqrt{(\underline{H}^T\underline{H})^{-1}{}_{xx} + (\underline{H}^T\underline{H})^{-1}{}_{yy} + (\underline{H}^T\underline{H})^{-1}{}_{zz} + (\underline{H}^T\underline{H})^{-1}{}_{tt}} = \sqrt{(\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 + \sigma_{tt}^2)/\sigma_r^2}$$

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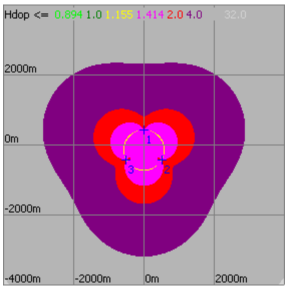
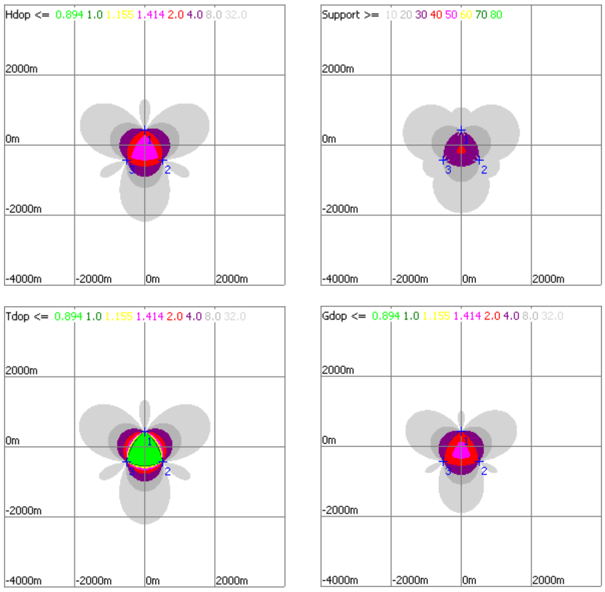
Absolute / Pseudo Range and Performance



Absolute Range ↑

Pseudo Range →



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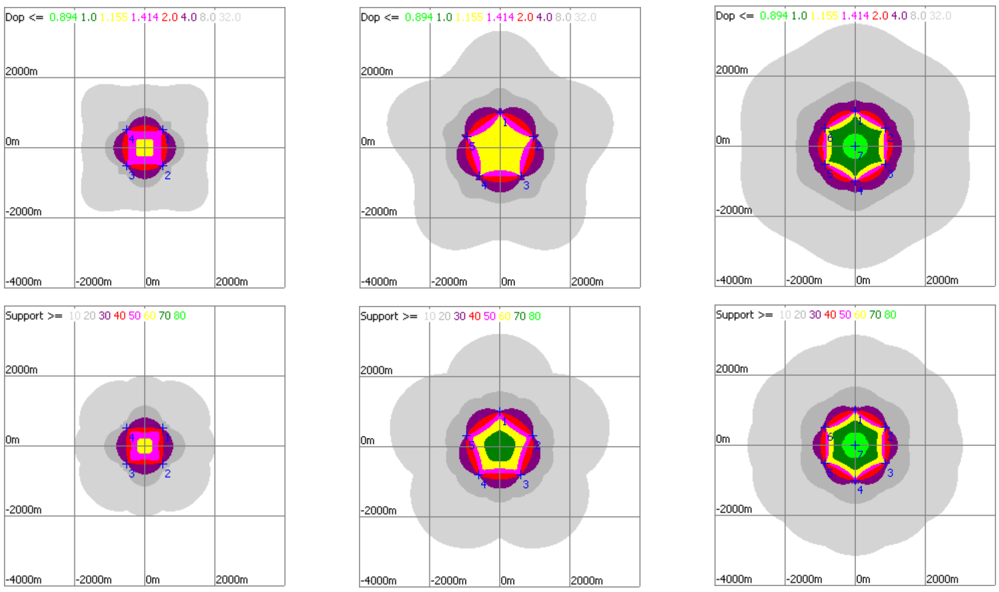
Hdop and Support Examples



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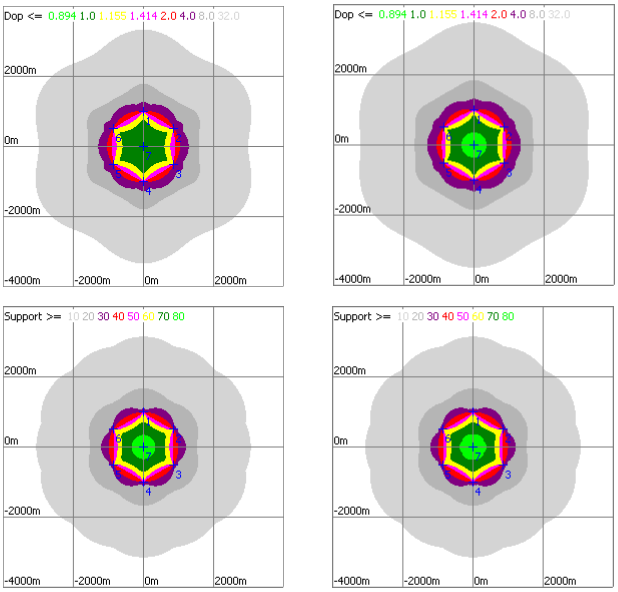
Hdop and Support Examples



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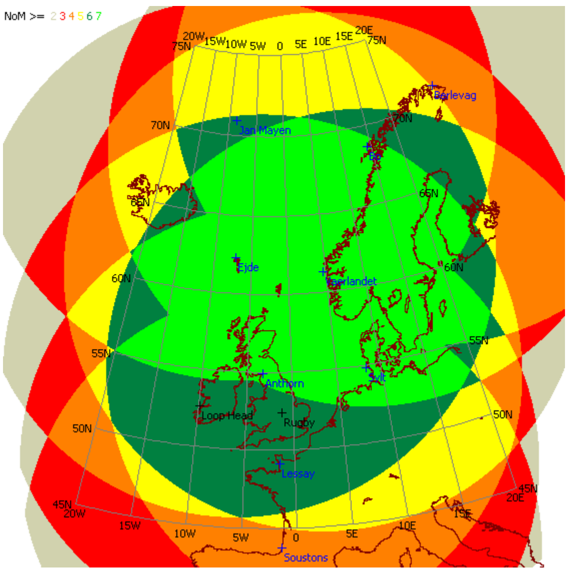
NW Europe : Loran Stations – Baseline : NoM



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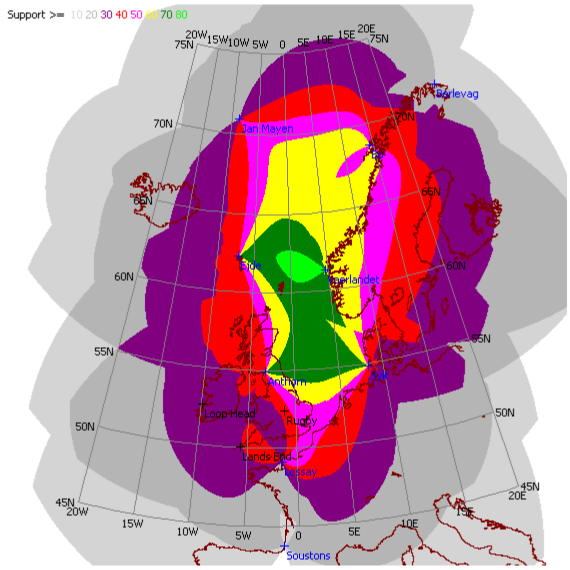
NW Europe : Loran Stations – Baseline : Support



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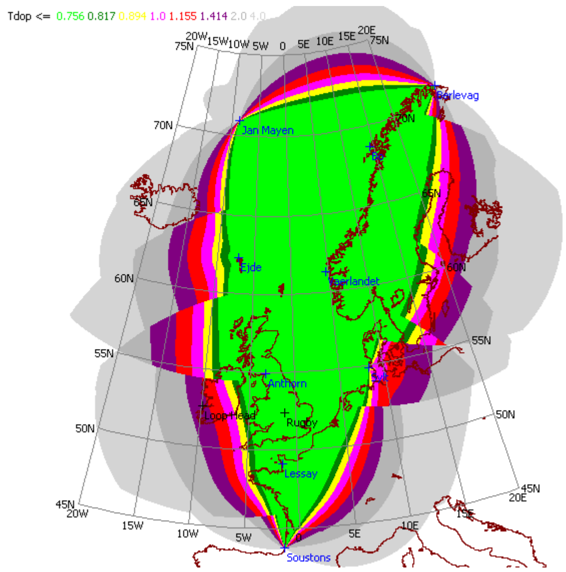
NW Europe : Loran Stations – Baseline : Tdop



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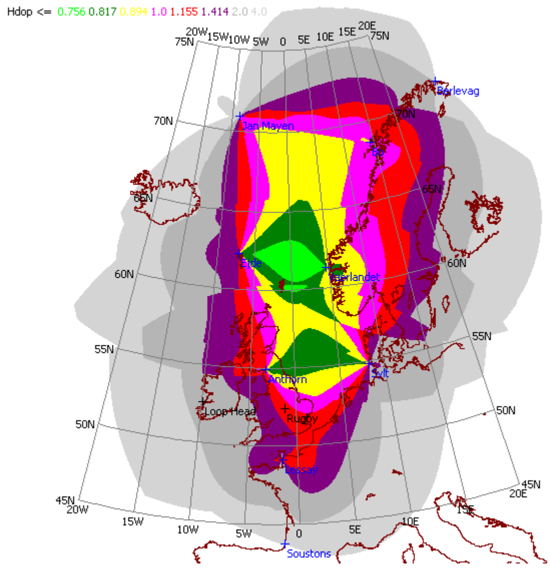
NW Europe : Loran Stations – Baseline : Hdop



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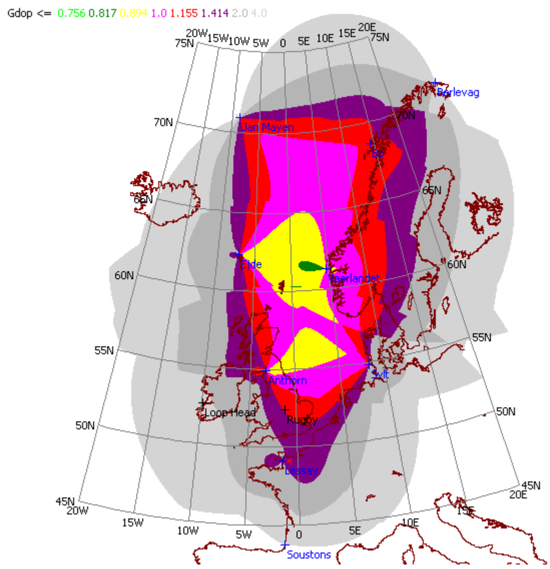
NW Europe : Loran Stations – Baseline : Gdop



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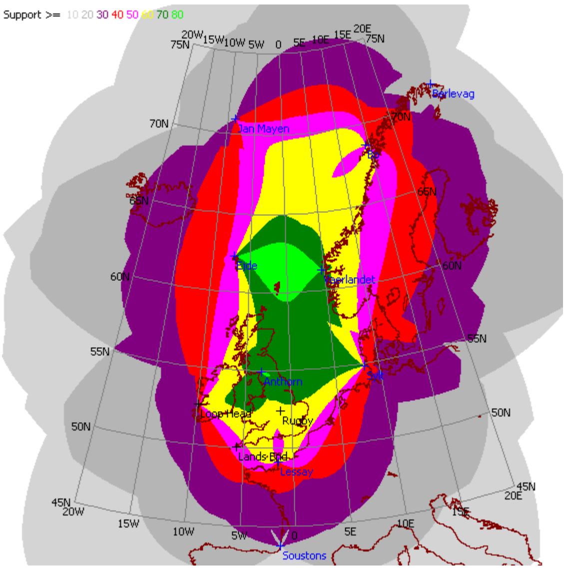
NW Europe : Loran Stations + Loop Head



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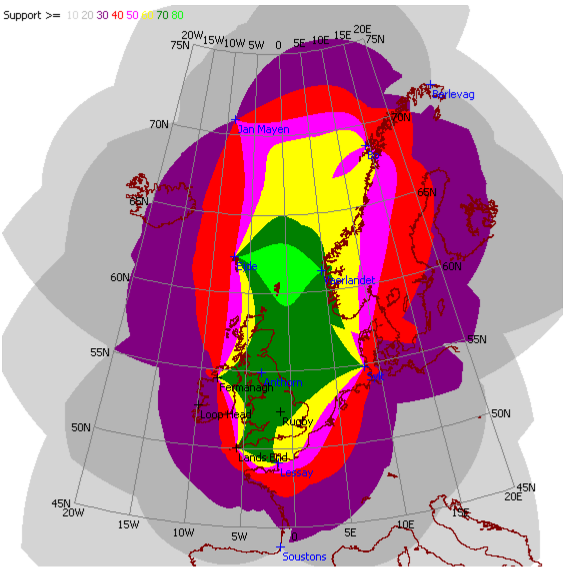
NW Europe : Loran Stations – A + F + LE



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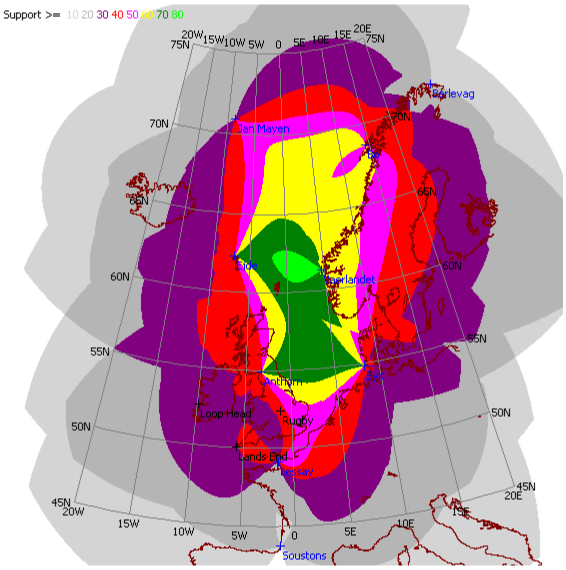
NW Europe : Loran Stations – Baseline : Support



Predicting Navigation Fix Accuracy

- Best Practice
  - Work (well) inside the sensor array
  - Perform support / DOP analysis
  - Assess, analyse and resolve second order issues
  - Post process system data for accuracy and insight
- Emeritus Solutions Ltd
  - Engineering consultancy
  - Underwater tracking range processing and analysis
  - Underwater acoustics and signal processing expertise
  - Instrumentation of signals, systems and computers
  - Software, hardware and system development

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Biography

Eur Ing Dr Martin Bishop is a Chartered Engineer and a Fellow of the Institution of Electrical Engineers. Martin is an Engineering Consultant with expertise assimilated during over two decades of involvement in defence R & D with the MoD, ultimately as a QinetiQ Fellow, and more recently in Offshore Oil and Gas. His current interests include Digital Signal Processing (from algorithms to implementations), platform design (from FPGAs through HDLs to digital hardware), embedded system design (from DSPs through microcontrollers to OOLs), underwater positioning (using trilateration techniques), underwater acoustics (from propagation to signal and system design), RF processing (including RFID) and the myriad uses of PCs.

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